**Week 4: 27/10/2021 – Wednesday**

1. **Outline of meeting**

The meeting was held on Zoom. The first half of the meeting was dedicated to bug-fixing the code from the previous week and correct any misunderstandings from the topic. The second half of the meeting was dedicated to the Mattis Bardeen Theory. A discussion was had regarding the topic and a careful elaboration to the best of my understanding was given. An outline of the Mattis Bardeen Theory will be elaborated in section 3 of this diary. Essentially, Mattis Bardeen theory is derived from the fundamental principles of superconductivity and takes the band gap into consideration. From this, the task that arose from this is to create the plots of Lint vs T using Mattis Bardeen Approximations to find the Lint from the band gap energy. Then, the resistive part of the impedance can be found, and a plot of R vs T can be made. The outline of the theory and task specification is given below.

1. **Specification of Tasks**
2. Plot out Lint over a temperature range of 0.05 – 4 K from Mattis Bardeen Approximations using equation 3.33 for a superconducting film with the following properties:

* Normal state conductivity:
* Thickness:
* Critical temperature:
* Frequency:

1. Use equation 3.38 to calculate R as a function of temperature
2. **Outline of Theory and Methodology for Task**

Mattis Bardeen Theory: *“The London Equations derived in previous weeks hold well but they are not derived from any fundamental principles of superconductivity and does not take into account the idea of a band gap. Another assumption of the London Model is that it assumes that the electrons in the superconducting state are just simply electrons which do not scatter and will all be accelerated independently if an electric field is applied.”*

*Source: Lumped Element Kinetic Inductance Detector – Dr. Simon Doyle Thesis*

Building from Mattis Bardeen Theory, the full effects of the band gap and non-local treatment of Cooper pairs leads to the Mattis Bardeen Integrals:

To simplify, the integrals can be approximated when in the limits and to the Mattis Bardeen Approximations:

Where is the band gap energy, I0 and k0 are modified Bessel functions of the first and second kind respectively. in this case was approximated to since .

Following this, the London Penetration depth can be found by first determining the electron density ns using the following relation:

Where = in this case. Using the result, the Penetration depth can be found:

Plugging into the expression for total internal inductance:

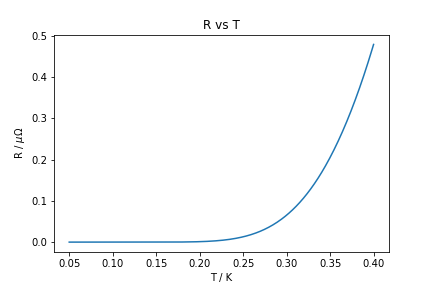
The graph of Lint vs T can be plotted.

Using the expression for the resistive part from equation 3.38 in the thesis:

A plot of R vs T can also be made. The plots and code for the tasks are shown in the following sections.

1. **Plot of L­­­­­­int vs T Using Mattis Bardeen ApproximationsChart

   Description automatically generated with low confidence**
2. **Plot of R vs T Using Mattis Bardeen Approximations**

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1. **Code for Calculating Lint and R**

#imports

import numpy as np

import matplotlib.pyplot as plt

from scipy.special import iv as I0

from scipy.special import kv as K0

import scipy.constants as const

#Define function

def coth(x):

return np.cosh(x)/np.sinh(x)

def csch(x):

return 1/np.sinh(x)

#Define constants

TC = 1.5

Delta\_0 = (3.5\*const.Boltzmann\*TC)/2

sigma\_n = 6.0e7 # Normal stae conductvity if superconducting film

Thick = 20e-9 # Thickness of superconducting fil

f = 500e6

w = 2 \* np.pi \* f

me = const.m\_e

miu\_0 = 4\*np.pi\*10\*\*-7

#Varying range of temperature

T = np.linspace(0.05, 0.4, num=500)

#An interpolation formula for delta\_T (Cheating a bit by using an interpolation formula, ideally should be integrated)

#Source: https://physics.stackexchange.com/questions/192416/interpolation-formula-for-bcs-superconducting-gap#mjx-eqn-eq2

delta\_T = Delta\_0\*np.tanh(1.74\*np.sqrt((TC/T)-1))

#Define constants to simplify eqn

multiplying\_constant = delta\_T/(const.hbar \* w)

e\_const\_1 = - Delta\_0/(const.Boltzmann\*T)

e\_const\_2 = (const.hbar\*w)/(2\*const.Boltzmann\*T)

#Parts of the sigma1 Ratio

A = 2\*multiplying\_constant

B = np.exp(e\_const\_1)

C = K0(0, e\_const\_2)

D = 2\*(np.sinh(e\_const\_2))

#Find Sigma 1 and Sigma 2

sigma1Ratio = A \* B \* C \* D

sigma2Ratio = np.pi\*multiplying\_constant\*(1 - (2\*np.exp(e\_const\_1)\*np.exp(-e\_const\_2)\*I0(0,e\_const\_2)))

sigma2 = sigma2Ratio \* sigma\_n

sigma1 = sigma1Ratio \* sigma\_n

# Sanity Check

# plt.subplot(2,1,1)

# plt.plot(T, sigma1Ratio)

# plt.ylabel("$\sigma\_{1}/\sigma\_n$")

# plt.yscale("log")

# plt.subplot(2,1,2)

# plt.plot(T, sigma2Ratio)

# plt.ylabel("$\sigma\_{2}/\sigma\_n$")

# plt.show()

# plt.figure()

#Depth

lower\_fraction = miu\_0\*sigma2\*w

Lambda\_T\_MB = (1/lower\_fraction)\*\*0.5

#Internal Inductance

fraction = Thick/(2\*Lambda\_T\_MB)

L\_int = (miu\_0\*Lambda\_T\_MB/2)\*coth(fraction)

plt.plot(T, L\_int\*10e12)

plt.ylabel("$L\_{int}$ / $pHSquare^{-1}$")

plt.xlabel("T / K")

plt.title("$L\_{int}$ vs Temperature")

plt.savefig("L\_int vs T Mattis Bardeen")

plt.figure()

#sigma 1 to sigma 2

sigma12Ratio = sigma1/sigma2

#Terms for lk

A = (miu\_0\*Lambda\_T\_MB)/4

B = coth(fraction)

C = fraction\*(csch(fraction))\*\*2

#R vs T

lk = A\*(B+C)

R = lk \* w \* sigma12Ratio

plt.title("R vs T")

plt.plot(T, R\*10\*\*6)

plt.ylabel("R / $\mu\Omega$")

plt.xlabel("T / K")

plt.savefig("R vs T Mattis Bardeen")